Enseigner les Mathématiques en Section Européenne. Exemples de pratique.
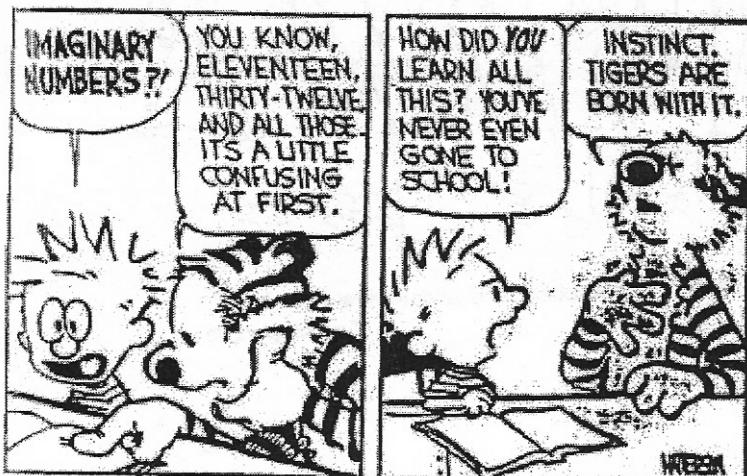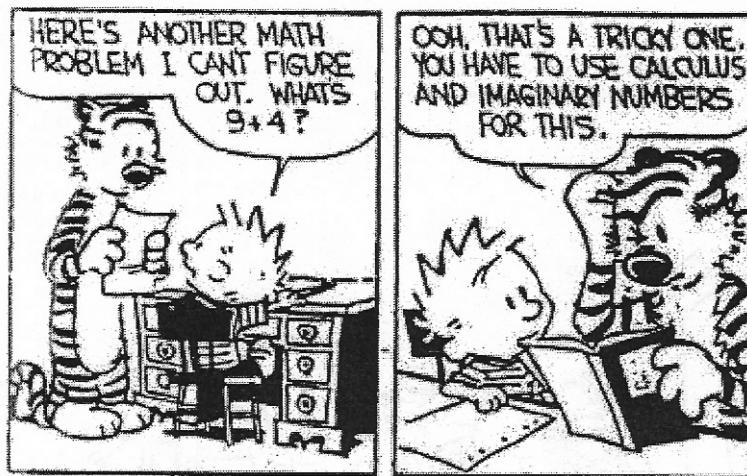
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Avant-propos

Ce document rassemble des activités menées en Section Européenne dans les classes de Seconde, Première et Terminale au lycée Alfred Kastler de Denain.

Il a pour but de répondre aux questions que se posent souvent les enseignants débutant en Section Européenne, au travers de conseils et d'exemples.

- Quels thèmes aborder en classe de Seconde ?
- Comment faire participer davantage les élèves ?
- Comment introduire une notion mathématique ?
- Où trouver des documents ? Comment les exploiter ?
- Comment évaluer les élèves ?
- Comment travailler avec le professeur de langue vivante ?

Mais voici avant tout quelques remarques qui ont guidé le choix et l'élaboration des exemples proposés.

Le premier objectif de cet enseignement est de favoriser un échange oral spontané en langue anglaise entre tous les acteurs de la classe, et la pratique des mathématiques s'y prête très bien à condition d'éviter certains écueils.

Tout d'abord, le cours de Section Européenne n'est pas une adaptation « exotique » du cours classique dispensé en France. Ce n'est pas non plus exactement le cours tel qu'il est dispensé en Angleterre.

En effet, la résolution d'exercices et la maîtrise des résultats du cours ne constituent pas en Section Européenne une fin en soi. Il s'agit davantage d'une autre façon de faire de l'anglais que d'une autre façon de faire des mathématiques. Les thèmes mathématiques abordés doivent donc être soigneusement choisis : effectuer de longs calculs ne présente pas un grand intérêt linguistique !

Il faut de plus garder à l'esprit qu'il s'agit d'une option, et qu'à ce titre elle doit rester attrayante : il est indispensable de varier les thèmes et les supports. L'utilisation de logiciels scientifiques en langue étrangère, l'exploitation de sites web, les voyages sont autant de façons de susciter l'intérêt des élèves.

Ensuite, le but de cet enseignement est clairement d'améliorer les capacités orales des élèves en langue étrangère : l'échange avec les élèves doit être vivant et de qualité. Il est donc préférable de se baser sur des documents authentiques, qui par leur forme et leur contenu vont susciter la prise de parole des élèves.

Mais au-delà de ce travail, l'enseignement en Section Européenne permet de valoriser les liens entre culture, langage et mathématiques, de faire comprendre aux élèves l'importance de la maîtrise de la langue dans le raisonnement mathématique et scientifique en général.

Cet enseignement constitue une véritable ouverture culturelle pour les élèves : les sites web de vulgarisation mathématique et scientifique, destinés à tous les niveaux , sont nombreux, de qualité, et offrent un large panorama sur la diversité des activités mathématiques et leur implication dans tous les secteurs.

Cela vaut aussi pour l'enseignant qui a le plaisir d'élargir le champ de son enseignement, d'organiser des séjours à l'étranger, de redécouvrir sa propre discipline à travers le prisme d'une autre langue, d'une autre culture. Et en mathématiques, les différences ne manquent pas.

Sommaire

Acquisition du vocabulaire spécifique

Certaines tournures ne s'improvisent pas et trouvent tout naturellement leur place dans le déroulement du cours et des exercices.

Mais attention ! Rappelons à nouveau que les résultats du cours, tout comme la résolution des exercices, ne constituent pas une fin en soi . **La pratique des mathématiques en Section Européenne sert avant tout de support à l'échange en langue étrangère**. C'est l'occasion pour les élèves de mettre en application leurs connaissances grammaticales tout en découvrant de nouvelles tournures de phrase.

Déroulement d'une séquence de cours dans cette optique.

Pour introduire une notion, on peut commencer par étudier plusieurs exemples, un énoncé d'exercice ou une situation problème: **un document authentique sert de point de départ.** Voici un exemple en Statistique pour la classe de seconde.

The marks obtained by 100 students in an examination were as shown below.

Marks	*0 – 5*	*6 - 10*	*11 - 15*	*16 - 20*
Frequency	*22*	*38*	*32*	*8*
Cumulative Frequency	*22*	*60*	*92*	*100*

For example, 60 students got 10 marks or less.
92 students got 15 marks or less.

What does the cumulative frequency mean ?
How can you find it ?

Puis, en plusieurs étapes, par questionnement et en explicitant en anglais le sens de certains termes, l'enseignant peut construire oralement avec les élèves une proposition.

The cumulative frequency is the total frequency up to a particular class boundary.
The cumulative frequency is found by adding each frequency to the sum of the previous ones.

On demande alors à un élève de la restituer. Ses camarades peuvent l'aider et lorsque tous sont d'accord (sur le fond et la forme), la définition est notée dans le cours.
Elle fera l'objet d'une brève interrogation orale lors du cours suivant.
Cette façon de procéder est très utile en classe de seconde pour trois raisons :
  1) du point de vue des mathématiques : la rigueur.
  2) du point de vue syntaxique : la richesse de la structure.
  3) du point de vue de la communication : la prise de parole de l'élève.
Et ce dernier point est essentiel : c'est de cette façon que les élèves s'approprient le vocabulaire, les tournures grammaticales et progressent à l'oral.

Voici un second exemple.

PRIME FACTORS

These are factors which are prime.
Some numbers can be written as products of their prime factors.

EXAMPLE
The diagram below shows the prime factors of 360.
• Divide 360 by its first prime factor, 2.
• Divide 180 by its first prime factor, 2.
• Keep on going until the final number is prime.
As a product of its prime factors 360 may be written as:

$2 \times 2 \times 2 \times 3 \times 3 \times 5 = 360$
or $2^3 \times 3^2 \times 5 = 360$

in index notation
(using powers).
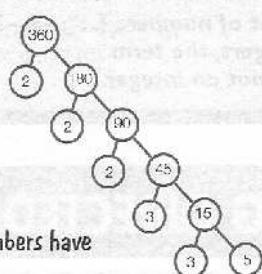
HIGHEST COMMON FACTOR (HCF)
The largest factor that two numbers have in common is called the HCF.

EXAMPLE
Find the HCF of 84 and 360.
• Write the numbers as products of their prime factors.
$84 = 2 \times 2 \times 3 \times 7$
$360 = 2 \times 2 \times 2 \times 3 \times 3 \times 5$
• Ring the factors in common
• These give the HCF = $2 \times 2 \times 3 = 12$

LOWEST COMMON MULTIPLE (LCM)
This is the lowest number which is a multiple of two numbers.

EXAMPLE
Find the LCM of 6 and 8.
• Write the numbers as products of their prime factors.
• $8 = 2 \times 2 \times 2$
• $6 = 2 \times 3$
• 8 and 6 have a common prime factor of 2. So it is only counted once.
• The LCM of 6 and 8 is $2 \times 2 \times 2 \times 3 = 24$

Knowledge and understanding.

1) What is a factor ? Give the factors of 20.
It's a whole number which divides exactly into another number, leaving no remainder.
2) What is a prime number ? Give the prime numbers up to 20.
It's a number which has only two factors : 1 and itself.
3) What does HCF mean ? Find the HCF of 24 and 60.
4) What does LCM mean ? Find the LCM of 24 and 60.

Certaines définitions et propositions doivent être connues parfaitement : la mémoire est sollicitée à bon escient car ces connaissances de base sont incontournables et constituent un bagage lexical et syntaxique dans lequel l'élève pourra puiser librement dès qu'il aura acquis un minimum d'autonomie.

De plus, l'élève ne se contente pas ainsi d'apprendre du vocabulaire technique, il mémorise des structures syntaxiques utiles dans tous les domaines.

Voici quelques exemples de propositions et de définitions qui peuvent être introduites de cette façon.

En classe de Seconde

En statistique :
-*A continuous variable is a variable which could take all possible values within a given range.*
-*A discrete variable is a variable which increases in steps.*
-*The term 'population' means everything or everybody in the category you are considering.*
-*Increasing an amount by 10% is the same as multiplying it by 1.1 .*
-*Data is the information you have obtained.*
-*The cumulative frequency is found by adding each frequency to the sum of the previous ones.*

Pour les transformations géométriques :
-A reflection moves an object so that it becomes a mirror image of itself .
-A rotation moves an object by turning it about a fixed point called the centre of rotation.
-To define a rotation, you need to state the position of the centre of rotation, the angle of rotation and the direction (clockwise or anticlockwise) of rotation.

Construction des droites remarquables du triangle :
-To bisect angle BAC : with A as centre, draw arcs to cut [AB) and [AC). With these two points as centres draw arcs to intersect each other. Join A to the point of intersection.
-To bisect the line (AB) : with A as centre, draw arcs above and below AB (the radius must be more than half AB). With B as centre, draw arcs to intersect the first arcs.

Pythagore et sa réciproque
-Pythagoras' theorem states that in any right-angled triangle, the square on the hypotenuse is equal to the sum of the squares on the other two sides.
-If the three sides of a triangle satisfy $a^2 = b^2 + c^2$, the angle opposite to the largest side is a right-angle.

En classe de Première

Les suites
-If we look at the sequence of numbers 2, 4, 6, 8, 10, … we can see that there are two ways of describing it. Either we notice that each number is two more than the previous one, or we notice that the value of any particular term is twice the value of its position.
-A sequence in which each term is found by multiplying the previous term by a fixed number is called a geometric sequence

Les probabilités
-The intersection of two sets A and B is the set of those things in common to both A and B.
-The union of two sets A and B is the set of those things which are either in A, or B, or both.
-A trial is an action with several possible outcomes.
-The set of possible outcomes for a trial is called the sample space.
-Any subset of the sample space is called an event.

En classe de Terminale

Fonctions et dérivation
- The slope of a continuous curve with equation $y = f(x)$ at any point on the curve is defined as the slope of the tangent to the curve at that point.
-A function f which increases as x increases in the interval from x=a to x=b is called an increasing function.
-The direction of variation depends on the sign of its derivative.
-The value of x which makes the function a maximum is 10.

Probabilités conditionnelles
-If you draw two cards from a pack with replacement, the result of the first selection doesn't affect the outcome of the second selection, so the first and the second draw are said to be independent.

-The solution where a piece of extra information is provided in a problem is referred to as a conditional probability.
-It is possible to show that if A is independent of B then conversely B is independent of A and this means that we can talk about independent event rather than the independence of one event from another.

Déroulement d'une séance d'exercices dans cette optique.

Le but est toujours de faciliter la prise de parole, et pour cela, les élèves doivent tout d'abord gagner en confiance.

Au début de l'année de seconde, lors d'une séance d'exercices, **on peut ainsi proposer un premier document authentique, présentant l'énoncé d'un exercice ainsi que sa résolution.** De tels documents se trouvent aisément dans les manuels anglais mais aussi sur internet.

On laisse les élèves le parcourir, s'imprégner de son contenu mathématique et linguistique. **Puis on propose un second énoncé, s'inspirant du premier.** On demande alors aux élèves de le résoudre, en s'aidant éventuellement du premier document.

Enfin, un élève est chargé d'exposer oralement sa solution. Lorsque tous les élèves sont d'accord, cette solution est notée.

Cette façon de procéder permet aux élèves de prendre de l'assurance, facilite ainsi la prise de parole et progressivement les élèves gagnent en autonomie.

Voici un exemple de document
Geometrical constructions.

Exercise A.
1) Construct a triangle GHI given the three lengths GH = 9 cm, GI = 6 cm and IH =8 cm.
2) Draw the perpendicular from point I to line (GH), using ruler and compass only.

Exercise B
Draw any triangle. Bisect the three angles of that triangle to show, by construction, that the angle bisectors all meet in the same point.

CIRCLE THEOREMS

There are several theorems you need to know and be able to apply.
The theorems are:

1. The perpendicular bisector of any chord passes through the centre.
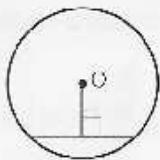
2. The angle in a semicircle is always 90°.
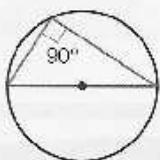
3. The radius and a tangent always meet at 90°.
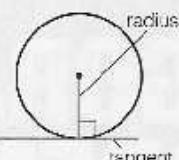
4. Angles in the same segment are equal, e.g. $A\hat{B}C = A\hat{D}C$

5. The angle at the centre is twice the angle at the circumference, e.g. $P\hat{O}Q = 2 \times P\hat{R}Q$
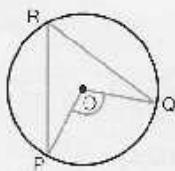
6. Opposite angles of a cyclic quadrilateral add up to 180°. (A cyclic quadrilateral is a 4-sided shape with each corner touching the circle), i.e. $x + y = 180°$
$a + b = 180°$
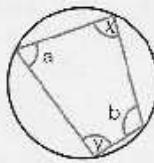
7. The lengths of two tangents from a point are equal, e.g. RS = RT
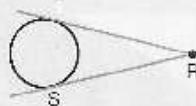
8. The angle between a tangent and a chord is equal to the angle in the alternate segment (that is the angle which is made at the edge of the circle by two lines drawn from the chord). This is known as the Alternate Segment Theorem. $a = b$.

Examiner's Top Tip
It is very important that you learn all of the eight theorems listed and that you can apply and explain ... the exam.
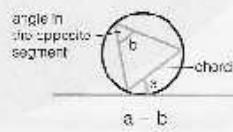
Un exemple de séance d'exercices en probabilités est proposé en page12 : *Tree diagrams*.

Intervention du professeur de langue vivante

Certaines structures spécifiques au discours mathématiques et scientifique en général peuvent être approfondies en collaboration avec les collègues de langue: comment exprimer l'objectivité, la comparaison, les dimensions, la cause et la conséquence, la fréquence. Comment articuler un raisonnement.

C'est d'ailleurs l'occasion de dépasser le cadre strict des mathématiques pour aborder les domaines économique, démographique, sociologique, biologique et élargir ainsi le champ lexical de l'élève tout en lui montrant des domaines d'application des mathématiques.

a) Exemples de thèmes de travail avec le professeur de langue vivante sur le vocabulaire spécifique.

Ces séances ont pour but la maîtrise des expressions et des termes indiqués en caractère gras.

Le professeur de langue vivante peut aisément, s'il le souhaite, introduire l'étude de ces thèmes lors de son enseignement, par le biais de textes ou d'exercices :

1) Measurement.

*This box is 25 cm **long** but it is only 13 cm **wide** and 3 cm **deep**.*
*The **average** weight of a baby at birth is 3.5 kg.*
***How high** is the Eiffel Tower ?*
*It is 10 cm **in diameter**.*
*The temperature **fell from** 20°C to 5°C.*
*The temperature **fell by** 15°C.*
*To count **up to** 30.*

2) Approximate measurements.

*See water freezes **at slightly under** 0°C.*
*It is **roughly** 5 cm long.*

3) Comparison and similarity.

*The Greenhouse effect could make the climate **hotter**.*
*The population growth in Mexico is **less than** that of other developing countries.*
*Light travels a million times **as fast as** sound.*
*The population of Japan in 2020 will be about the **same as** it was in 1987.*
*Japan has the **lowest** population growth rate.*
*The **worst** drivers are under 20.*

4) Link words.

*Amphetamines are dangerous, **therefore** they should be kept away from children.*
*Water usually boils at 100°C, **however** at higher altitudes, the temperature is lower.*
*There was a short circuit, **as a result** the light went out.*

*Mercury is a metal, **nevertheless** it is a fluid.*
***As a rule**, water freezes at 0°C but it doesn't do so if you add salt.*

5) Cause and consequence.

***Thanks to** chlorophyll, CO_2 is transformed into oxygen.*
*Observers had difficulties identifying the comet, **because of** its speed.*
*Forests will disappear, **unless** fewer trees are cut down for fuelwood.*
***Provided** the tests show that the drug is safe, clinical trials can begin next month.*

6) Impersonal forms and modals.

*Radioactivity **can be** detected by a Geiger counter.*
*Life **might be** discovered on Mars.*
*The Earth **is heated** by the sun.*

b) La préparation de l'examen.

Le professeur de langue vivante est plus à même de corriger les erreurs de prononciation des élèves et de consolider leurs connaissances grammaticales.

Son apport est donc précieux dans la préparation de l'épreuve du baccalauréat.

La seconde partie de l'épreuve de Section Européenne est en effet un entretien mené par un professeur de langue vivante.

L'élève est essentiellement jugé sur ses capacités orales de communication mais la qualité de la prononciation et la rigueur grammaticale sont bien sûr valorisées : on attend d'ailleurs de l'élève qu'il parvienne au niveau B2 du Cadre Européen des Langues Vivantes.

Utilisation d'Internet

Revenons sur l'importance de travailler sur des documents authentiques, quel que soit le support : seuls ces documents permettent une réelle immersion et donnent du sens à l'enseignement dispensé en Section Européenne.

Internet permet de se procurer simplement et gratuitement de tels documents.

Cours de mathématiques en ligne, énigmes et jeux mathématiques, biographies de mathématiciens célèbres, exercices interactifs, sites de vulgarisation scientifique et mathématique : les ressources ne manquent pas et permettent de renouveler constamment l'intérêt des élèves.

Deux types de séances sont envisageables :

Si l'on vise le réinvestissement du vocabulaire spécifique au niveau seconde et première :

De nombreux sites anglais proposent de réviser en ligne et gratuitement les connaissances indispensables à l'obtention du GCSE.

On peut y trouver en ligne le cours de mathématiques ainsi que de nombreux exercices d'application.

Les élèves peuvent travailler en autonomie sur ces sites.

http://www.bbc.co.uk/schools/gcsebitesize/maths/
http://www.gcseguide.co.uk/mathsgcseguide.htm

Si l'on souhaite travailler sur un document à contenu mathématique ou scientifique :
http://www.sciencenewsforkids.org/
http://en.wikipedia.org/wiki/Pythagorean_theorem
http://www.mathsisfun.com/
http://www-history.mcs.st-andrews.ac.uk/

Dans ce cas, on choisit un article, on laisse les élèves lire le document en intégralité sans apporter d'aide. On évite l'usage de dictionnaires.

On demande alors aux élèves de donner à l'oral en anglais les idées principales développées dans le document.

Puis on procède à une lecture à voix haute, qu'on n'hésite pas à interrompre pour expliciter en anglais le sens de certains termes.

Enfin, on peut prévoir un questionnaire que chaque élève remplit seul, permettant de s'assurer qu'il a bien compris le document : cette trace écrite sera conservée et éventuellement ré exploitée.

Voici quatre exemples de documents tirés des sites indiqués ci-dessus. Chacun d'entre eux est accompagné d'un questionnaire destiné aux élèves.

Exemple 1: Setting a prime number record.　　(extrait de Science News for Kids)

Commentaire : ce texte, <u>qui peut faire l'objet de deux séances</u>, permet d'aborder de façon originale l'arithmétique en langue anglaise. De nombreuses définitions sont données et le problème de la recherche de grands nombres premiers clairement expliquée.

First part.

What's the biggest number you can think of? A billion? A trillion? A googol? (That's 1 followed by 100 zeroes.) Whatever number you come up with, there's always a larger one. You could write down 1 and keep adding zeroes after it until you hand gets tired, and you still wouldn't get to the "last" number. There's always another number right after whatever you've written down. Just add 1 and you'll get a bigger number.

Certain types of numbers, though, are special. A computer search has now turned up the largest example yet found of a type known as prime numbers. The new champion is 7,816,230 digits long. If you could write 10 digits per second, it would take you more than 9 days to copy out the entire number!

There are lots of different kinds of numbers and lots of ways to play with these numbers. You may have learned about whole numbers, fractions, integers, or even imaginary numbers. And you've probably done plenty of adding and subtracting, multiplying and dividing.

For most people, math is just a useful set of skills that helps them count, make change, or cut cakes and pizzas into pieces of the right size or shape. For mathematicians and others, however, there's something magical about numbers, especially those that fall into the category of prime numbers.

Second part.

Prime numbers are whole numbers that can be divided evenly only by themselves and 1. One example of a prime number is 13. Only the numbers 1 and 13 divide into 13 without leaving a remainder. The number 8 is not a prime because it's divisible by 1, 2, 4, and 8—not just 1 and 8. The first few prime numbers are 2, 3, 5, 7, 11, 13, 17, 19, 23, and 29.

It can be really hard to tell whether a number is a prime, especially if it's a huge number. It's easy to check whether 13 is a prime, for example. Just divide all the numbers that come before it into 13 and make sure that none divide into 13 evenly. For a big number, even with all sorts of shortcuts that mathematicians have found over the years for doing this, it takes much, much longer to find out.

The new champion prime was found by a computer. As part of the Great Internet Mersenne Prime Search (GIMPS), people all over the world have donated computer time to search for primes. More than 250,000 computers are now involved, each one looking for primes whenever someone isn't using the computer for anything else.

The record-breaking number turned up on an office computer owned by Martin Nowak, a German eye surgeon and a huge math fan. At 7,816,230 digits, it's some 500,000 digits longer than the previous record holder. The number can also be written as 2 to the 25,964,961st power minus 1. That's one less that 2 multiplied by itself 25,964,961 times. Try that on your calculator!

Now, computers worldwide are looking for an even bigger prime. And there's no end in sight. There will always be a bigger one!—*E. Sohn*

<u>Now test yourself</u>
1) What's this text about ?
2) What is a prime number ? Give examples.
3) What record does this text refer to ?
4) How can you check that a given number is prime ?
5) What does GIMPS mean ?Who is Martin Nowak ?

Exemple 2 : Reflections

Commentaire : un exemple de cours disponible sur Internet. L'élève peut vérifier « en conditions réelles » sa maîtrise du vocabulaire spécifique.

Remember

The object and its image are always the same perpendicular distance from the mirror line. Perpendicular means 'at right angles to'.

Examples:
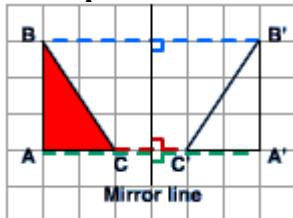
A and A' are the same perpendicular distance from the mirror line, as are B and B' and C and C'

The object (ABC) has been **reflected** in the mirror line to give the image (A'B'C').
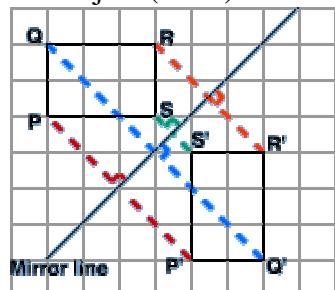
P and P' are the same perpendicular distance from the mirror line, as are Q and Q', R and R' and S and S'.

The object (PQRS) has been **reflected** in the mirror line to give the image (P'Q'R'S').

Example 3 : Tree diagrams

Commentaire : ce document destiné aux élèves de première reprend toutes les notions du cours de probabilité et constitue une première approche des sujets du baccalauréat.

Tree diagrams allow us to see all possible outcomes of an event and calculate their probabilities. Each branch in a tree diagram represents a possible outcome for an event.
If two events are **independent**, then the outcome of one has no effect on the outcome of the other. For example, if we toss two coins, getting a head with the first coin will not affect the probability of getting a head with the second coin.
Three coins are tossed. What is the probability of getting:
A. three heads?
B. two heads and a tail?
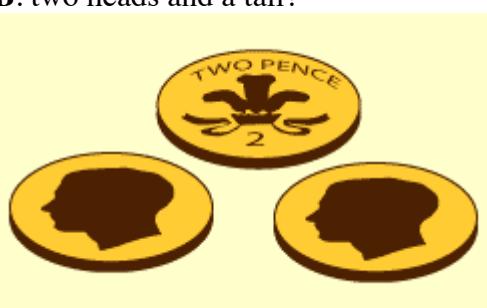
The Answer
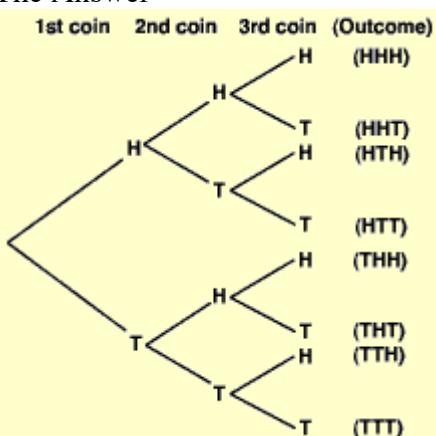
From the tree diagram we can see that there are eight possible outcomes. To find out the probability of a particular outcome we need to look at all the available paths (set of branches) in the tree diagram.

A. Only one path has three heads, so the probability of getting three heads is $\frac{1}{8}$
B. Three of the outcomes show two heads and a tail, so the probability of getting two heads

and a tail is $\frac{3}{8}$
This method is very effective when there are only a few possible outcomes and when the events are independent, but in most cases you will need to calculate the answer using probabilities.

Look at the tree diagram again, this time with the probabilities written on each branch:

$$P(HHH) = \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{8}$$

$$P(2\text{ Heads and a Tail}) = P(HHT) + P(HTH) + P(THH)$$

$$= \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} + \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} + \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2}$$

$$= \frac{1}{8} + \frac{1}{8} + \frac{1}{8}$$

$$= \frac{3}{8}$$
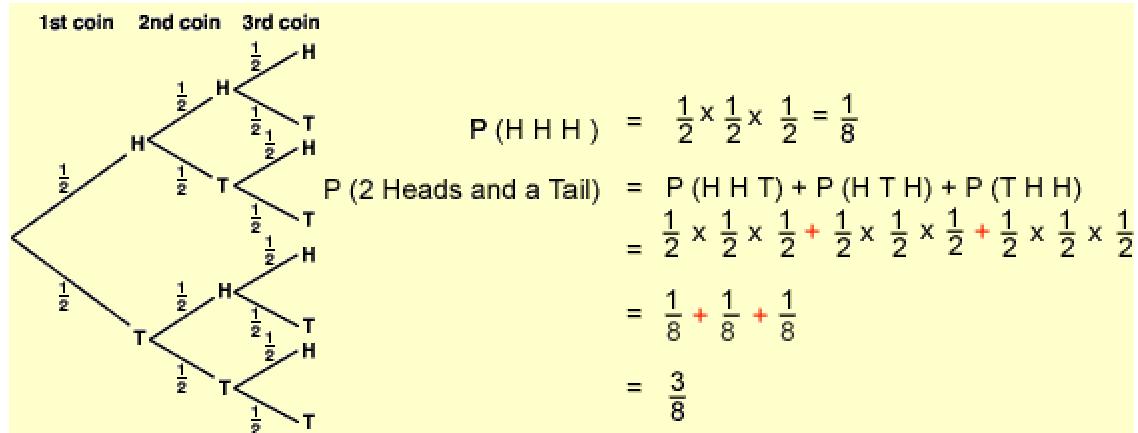
Remember the sum of the probabilities for any set of branches should always be 1, in this example there is a 0.5 chance of getting either heads or tails and 0.5 + 0.5 = 1.
Remember that in a tree diagram we **multiply horizontally** and **add vertically**.

Now test yourself

Exercise 1

I stand a 75% chance of passing my driving test.
If I fail, I stand a 90% chance of passing next time.
Whatever happens, I am not going to pass it more than twice.
By drawing a tree diagram, find the probability that I pass.

Exercise 2

Two cards are drawn randomly from an ordinary pack of 52 playing cards without replacement. Find the probability that :
  a) The two cards are of the same colour.
  b) Just one of the cards is a spade.

Exemple 4 : the Pythagorean theorem.   (extrait de Wikipedia)

Commentaire : ce document propose plusieurs démonstrations en langue anglaise du théorème de Pythagore. Toutes sont abordables dés la classe de seconde.
Dans un premier temps, le théorème lui-même est rappelé.

From Wikipedia, the free encyclopedia

In mathematics, the **Pythagorean theorem** or **Pythagoras' theorem** is a relation in Euclidean geometry among the three sides of a right triangle. The theorem is named after the Greek mathematician Pythagoras, who by tradition is credited with its discovery and proof,[1] although knowledge of the theorem almost certainly predates him. The theorem is known in mainland China as the "Gougu theorem" for the $(3, 4, 5)$ triangle.
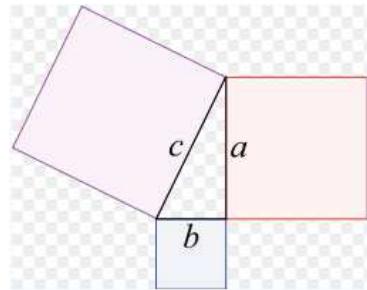
The Pythagorean theorem: The sum of the areas of the two squares on the legs (a and b) equals the area of the square on the hypotenuse (c).
The theorem is as follows:
In any right triangle, the area of the square whose side is the hypotenuse (the side opposite the right angle) is equal to the sum of the areas of the squares whose sides are the two legs (the two sides other than the hypotenuse).
This is usually summarized as:
The square on the hypotenuse is equal to the sum of the squares on the other two sides.
If we let c be the length of the hypotenuse and a and b be the lengths of the other two sides, the theorem can be expressed as the equation
$$a^2 + b^2 = c^2$$
or, solved for c:
$$\sqrt{a^2 + b^2} = c.$$
This equation provides a simple relation among the three sides of a right triangle so that if the lengths of any two sides are known, the length of the third side can be found.

Now test yourself

    1) *Who is Pythagoras ?*
    2) *What does Pythagoras' theorem state ?*
    3) *Can it be used in any triangle ?*
    4) *What is that theorem used for ?*

Proof using similar triangles
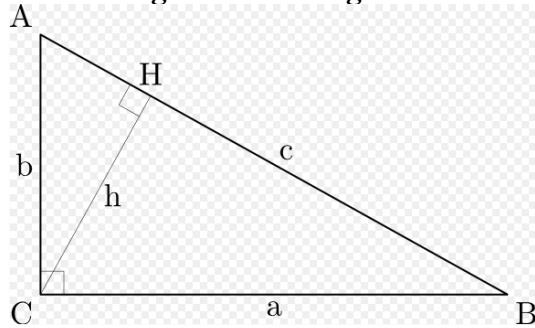
Like many of the proofs of the Pythagorean theorem, this one is based on the proportionality of the sides of two similar triangles.

Let *ABC* represent a right triangle, with the right angle located at *C*, as shown on the figure. We draw the altitude from point *C*, and call *H* its intersection with the side *AB*. The new triangle *ACH* is similar to our triangle *ABC*, because they both have a right angle (by definition of the altitude), and they share the angle at *A*, meaning that the third angle will be the same in both triangles as well. By a similar reasoning, the triangle CBH is also similar to *ABC*. The similarities lead to the two ratios:

As

$$BC = a, AC = b, \text{ and } AB = c,$$

so

$$\frac{a}{c} = \frac{HB}{a} \text{ and } \frac{b}{c} = \frac{AH}{b}.$$

These can be written as

$$a^2 = c \times HB \text{ and } b^2 = c \times AH.$$

Summing these two equalities, we obtain

$$a^2 + b^2 = c \times HB + c \times AH = c \times (HB + AH) = c^2.$$

In other words, the Pythagorean theorem:

$$a^2 + b^2 = c^2.$$

Knowledge and understanding.

1) Can you quote Pythagoras'theorem ?
2) Explain what similar triangles are.
3) What is the altitude of a triangle ?
4) Prove that the triangle CBH is similar to ABC.
5) Read again that document carefully and try to explain it to the class.

Le texte qui suit, plus long et d'un contenu plus difficile, expose la démonstration du théorème de Pythagore élaborée par Euclide. Ce texte, à étudier en classe, peut faire l'objet de deux séances.

Euclid's proof
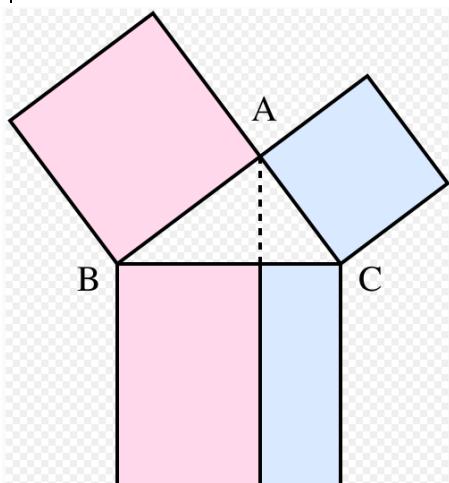
In Euclid's *Elements*, the Pythagorean theorem is proved by an argument along the following lines. Let A, B, C be the vertices of a right triangle, with a right angle at A. Drop a perpendicular from A to the side opposite the hypotenuse in the square on the hypotenuse. That line divides the square on the hypotenuse into two rectangles, each having the same area as one of the two squares on the legs.

For the formal proof, we require four elementary lemmata:

If two triangles have two sides of the one equal to two sides of the other, each to each, and the angles included by those sides equal, then the triangles are congruent.

The area of a triangle is half the area of any parallelogram on the same base and having the same altitude.

The area of any square is equal to the product of two of its sides.

The area of any rectangle is equal to the product of two adjacent sides (follows from Lemma 3).

The intuitive idea behind this proof, which can make it easier to follow, is that the top squares are morphed into parallelograms with the same size, then turned and morphed into the left and right rectangles in the lower square, again at constant area.

The proof is as follows:

Let ABC be a right-angled triangle with right angle CAB.

On each of the sides BC, AB, and CA, squares are drawn, CBDE, BAGF, and ACIH, in that order.

From A, draw a line parallel to BD and CE. It will perpendicularly intersect BC and DE at K and L, respectively.

Join CF and AD, to form the triangles BCF and BDA.

Angles CAB and BAG are both right angles; therefore C, A, and G are colinear. Similarly for B, A, and H.

Angles CBD and FBA are both right angles; therefore angle ABD equals angle FBC, since both are the sum of a right angle and angle ABC.

Since AB and BD are equal to FB and BC, respectively, triangle ABD must be equal to triangle FBC.

Since A is colinear with K and L, rectangle BDLK must be twice in area to triangle ABD.

Since C is colinear with A and G, square BAGF must be twice in area to triangle FBC.

Therefore rectangle BDLK must have the same area as square BAGF = AB^2.

Similarly, it can be shown that rectangle CKLE must have the same area as square ACIH = AC^2.

Adding these two results, $AB^2 + AC^2$ = BD BK + KL KC

Since BD = KL, BD BK + KL KC = BD(BK + KC) = BD BC

Therefore $AB^2 + AC^2 = BC^2$, since CBDE is a square.

This proof is mentioned in Euclid's *Elements* as proposition 1.47.

1) List the link words mentioned on that text.
2) What is a lemma ?
3) Explain each of this lemma in your own words.
4) What does 'colinear' mean ?
5) Explain Euler's intuitive idea to prove Pythagoras' theorem.

Algebraic proof
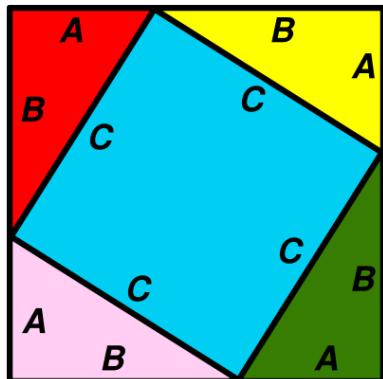
This is a square created by aligning four right angle triangles and a large square.
An algebraic variant of this proof is provided by the following reasoning. Looking at the illustration which is a large square with identical right triangles in its corners, the area of each of these four triangles is given by

$$\frac{1}{2}AB.$$

The *A*-side angle and *B*-side angle of each of these triangles are <u>complementary angles</u>, so each of the angles of the blue area in the middle is a right angle, making this area a square with side length *C*. The area of this square is C^2. Thus the area of everything together is given by:

$$4\left(\frac{1}{2}AB\right) + C^2.$$

However, as the large square has sides of length *A* + *B*, we can also calculate its area as
$(A + B)^2$, which expands to $A^2 + 2AB + B^2$.

$$A^2 + 2AB + B^2 = 4\left(\frac{1}{2}AB\right) + C^2.$$

(Distribution of the 4) $A^2 + 2AB + B^2 = 2AB + C^2$

(Subtraction of 2*AB*) $A^2 + B^2 = C^2$

Knowledge and understanding

1) What are complementary angles ?
2) Prove that the blue area in the middle is a square. .
3) Read again that document carefully and try to explain it to the class.

Exemples de travaux sur des textes authentiques

Ces textes, rédigés dans un anglais plus littéraire, offrent un alternative aux documents fournis par Internet: On ne vise plus seulement ici l'amélioration des capacités de communication mais plutôt la formation d'une culture anglo-saxonne, scientifique et historique solide.

Ces textes sont très riches, tant au niveau de la langue que du contenu mathématique, et sont destinés à des activités d'approfondissement : on les traitera plutôt lors du troisième trimestre en entraînant d'abord les élèves sur des documents plus simples issus d'Internet.

Ces textes ont été choisis car ils illustrent les rapports entre les mathématiques, la culture et le langage : toutes les compétences développées en Section Européenne sont sollicitées. Chaque texte est accompagné d'un questionnaire afin d'en vérifier la compréhension et de guider le travail des élèves.

Chaque élève lit d'abord seul le texte dans sa globalité, ainsi que les questions proposées. Durant cette première partie, l'élève est seul face au texte et tente de deviner le sens des termes qu'il ne connaît pas. On effectue alors un premier bilan oral sur le texte et les questions.

Puis l'enseignant reprend oralement la lecture, n'hésitant pas à l'interrompre pour expliquer en anglais le sens de certains termes.

Chaque élève répond alors par écrit aux questions posées.

On invite plusieurs élèves à lire leurs réponses.

On effectue une synthèse avec la classe. Le résultat définitif est noté. Cette trace écrite pourra éventuellement être réexploitée.

Premier exemple (niveau seconde)

Commentaire : il s'agit d'une introduction à la cryptographie, que l'on peut planifier sur six séances consécutives. Plusieurs types de codage y sont décrits ainsi que leurs avantages et inconvénients respectifs. Le premier texte commence par une mise en perspective historique du développement de la cryptographie.

The evolution of secret writing (Simon Singh, The code book)

1st part

> Secret communication achieved by hiding the existence of a message is known as *steganography*, derived from the Greek words *steganos*, meaning 'covered' and *graphein*, meaning 'to write'. Various forms of steganography have been used throughout the world. For example, as far back as the first century AD, Pliny the Elder explained how the milk of a plant could be used as an invisible ink.
>
> But steganography suffers from a fundamental weakness : if the message is discovered, then the contents of the secret communication are revealed at once. Interception of the message immediately compromises all security.
>
> Hence, in parallel with the development of steganography, there was the evolution of cryptography, derived from the Greek word *kryptos*, meaning 'hidden'.

> The aim of cryptography is not to hide the existence of a message, but rather to hide its meaning, a process known as *encryption*.
>
> To render the message unintelligible, it is scrambled according to a particular protocol which is agreed beforehand between the sender and the intended recipient.
>
> The advantage of cryptography is that if the enemy intercepts an encrypted message, the message is unreadable. Without knowing the scrambling protocol, the enemy should find it difficult, if not impossible, to recreate the original message from the encrypted text.
>
> Of the two branches of secret communication, cryptography is the more powerful because of this ability to prevent information from falling into enemy hands.

Tool box.

To scramble   /   a weakness   /   to prevent

Using the source.
1) *What is steganography ? Give an example.*
2) *Is it a reliable system ?*
3) *What does cryptography consist in ?How does it work ?*
4) *Why is it more powerful than steganography ?*

2nd part

> Cryptography itself can be divided into two branches, known as *transposition* and *substitution*.
>
> In transposition, the letters of the message are simply rearranged, effectively generating an anagram. For very short messages, such as a single word, this method is relatively insecure because there are only a limited number of ways of rearranging a handful of letters. For example, three letters can be arranged in only six different ways, e.g. **cow**, **cwo**, **ocw**, **owc**, **wco**, **woc**.
>
> However, as the number of letters gradually increases, the number of possible arrangements rapidly explodes, making it impossible to get back to the original message unless the exact scrambling protocol is known.
>
> **For example, consider this short sentence.** It contains just 35 letters, and yet there are more than 50,000,000,000,000,000,000,000,000,000,000 distinct arrangements of them. If one person could check one arrangement per second, and if all the people in the world worked night and day, it would still take more than a thousand time the life of the universe to check all the arrangements.
>
> But there is a drawback. Transposition effectively generates an incredibly difficult anagram, even for the intended recipient. In order for transposition to be effective, the rearrangement of letters needs to follow a system, one that has been previously agreed by sender and receiver , but kept secret from the enemy.

Tool box.

a drawback   /   a handful

Using the source.

1) *What does transposition mean ?*
2) *What happens if the messge is short ? Is it safe ?*
3) *What happens if the message is too long ?*

The alternative to transposition is substitution. One of the recommended techniques is to pair letters of the alphabet at random, and then substitute each letter in the original message with its partner.

If we apply the principle to the Roman alphabet, we could pair letters as follows:

A	D	H	I	K	M	O	R	S	U	W	Y	Z
|	|	|	|	|	|	|	|	|	|	|	|	|
V	X	B	G	J	C	Q	L	N	E	F	P	T

Then , instead of **meet at midnight**, the sender would write **CUUZ VZ CGXSGIBZ**.

This form of secret writing is called a substitution cipher because each letter in the text is substituted for a different letter.

In transposition, each letter retains its identity but changes its position, whereas in substitution each letter changes its identity but retains its position.

The first documented use of a substitution cipher for military purposes appears in Julius Caesar's *Gallic Wars* .

Caesar describes how he sent a message to Cicero, who was besieged.

He simply replaced each letter in the message with the letter that is three places further down the alphabet.

Original alphabet	a	b	c	d	e	f	g	h	i	j …
Cipher alphabet	D	E	F	G	H	I	J	K	L	M …

Tool box.

a cipher   /   a purpose   /   besieged

Using the source.

1) What does substitution consist in ?
2) Compare transposition and substitution.
3) Describe Caesar's cipher.

Second exemple (niveau première)

Commentaire : le rôle du zéro en mathématiques et l'abandon des chiffres romains.

The evolution of number theory (Simon Singh, Fermat's last theorem)

In modern mathematics zero performs two functions.

First, it allows us to distinguish between numbers like 52 and 502. In a system where the position of a number denotes its value, a symbol is needed to confirm an empty position. For instance, 52 represents 5 times ten plus 2 times one, whereas 502 represents 5 times a hundred plus 0 times ten plus 2 times one, and the zero is crucial for removing any ambiguity. Even the Babylonians in the third millennium BC appreciated the use of zero to avoid confusion, and the Greeks adopted their idea, using a circular symbol similar to the one we use today.

However, zero has a more subtle and deeper significance which was only appreciated several centuries later by the mathematicians of India. The Hindus recognised that zero had an independent existence beyond the mere spacing role among the other numbers : zero was a number in its own right. It represented a quantity of nothing.

For the first time, the abstract concept of nothingness had been given a tangible symbolic representation.

This may seem a trivial step forward to the modern reader, but the deeper meaning of the zero symbol had been ignored by all the ancient Greek philosophers, including Aristotle.

He had argued that the number zero should be outlawed because it disrupted the consistency of the other numbers : dividing any ordinary number by zero led to an incomprehensible result.

As well as adding zero to the mathematical vocabulary, India and Arabia replaced the primitive Greek symbols and Roman numerals with the counting system which has now been universally adopted.

Once again, this might seem like an absurdly humble step forward, but try multiplying CLV by DCI and you will appreciate the significance of the breakthrough : the equivalent task of multiplying 155 by 601 is a good deal simpler.

The growth of any discipline depends on the ability to communicate and develop ideas, and this in turn relies on a language which is sufficiently detailed and flexible.

Tool box.

To perform   /   to disrupt   /   to avoid   /   a breackthrough

Knowledge and understanding.

1) What is zero used for in modern mathematics ?
2) Why did Aristotle reject the use of zero ?
3) Why were the Roman numerals replaced ?

Troisième exemple (niveau terminale)

Commentaire : le premier texte, après être revenu sur le travail d'Euclide, décrit le principe du raisonnement par l'absurde.

Text 1

The evolution of number theory (Simon Singh , Fermat's last theorem)

Euclid was born in about 330 BC. Like Pythagoras, Euclid believed in the search for mathematical truth for its own sake and did not look for applications of his work.

One story tells of a student who questioned him about the use of the mathematics he was learning. Euclid turned to his slave and said, 'Give the boy a penny since he desires to profit from all that he learns'. The student was then expelled.

Euclid devoted much of his life to writing the *Elements*.

The *Elements* consist in thirteen books, some which are devoted to Euclid's own work, and the remainder being a compilation of all the mathematical knowledge of the age, including two volumes devoted entirely to the works of the Pythagorean Brotherhood.

In the centuries since Pythagoras, mathematicians had invented a variety of logical techniques which could be applied in different circumstances, and Euclid skilfully employed them all in the *Elements*.

In particular Euclid exploited a logical weapon known as *reductio ad absurdum*, or proof by contradiction. The approach revolves around the perverse idea of trying to prove that a theorem is true by first assuming that the theorem is false. The mathematician then explores the logical consequences of the theorem being false.

At some point along the chain of logic there is a contradiction (e.g. $2 + 2 = 5$) .

Mathematics abhors a contradiction and therefore the theorem cannot be false, i.e. it must be true.

Tool box.

Sake   /   skilfully   /   to abhor

Using the source.

1) Who is Euclid ? Why is he so famous ?
2) What do the Elements consist in ?
3) Describe in your own words what a proof by contradiction is.

Text 2
Commentaire : ce second texte est une introduction à caractère historique de la théorie des graphes. Il propose de plus la démonstration d'un premier résultat.

Graph theory (Simon Singh , Fermat's last theorem)

A problem which also appealed to Euler's whimsical nature concerned the Prussian city of Königsberg, now known as the Russian city of Kaliningrad.
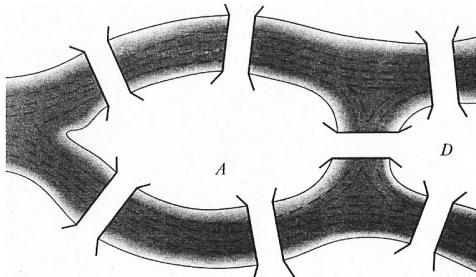
The city is built on the banks of the river Pregel and consists of four separate quarters connected by seven bridges. Some of the more curious residents of the city wondered if it was possible to make a journey such that each bridge is crossed once, and only once.

The citizens of Königsberg tried various routes but each one ended in failure.

Euler also failed to find a successful route, but he was successful in explaining why making such a journey was impossible.

Euler began with a plan of the city, and from it he generated a simplified representation in which the sections of land were reduced to points and bridges were replaced by lines.
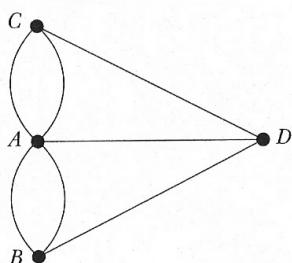
He then argued that, in general, to make a successful journey (i.e. crossing all the bridges only once) a point should be connected to an even number of lines.

This is because in the middle of a journey when the traveller passes through a land mass, he or she must enter via one bridge and then leave via a different bridge.

There are only two exceptions to this rule : when a traveller either begins or ends the journey.

So, in general, Euler concluded that for any network of bridges, it is only possible to make a complete journey crossing each bridge only once if all the land masses have an even number of bridges, or exactly two land masses have an odd number of bridges.

In the case of Königsberg, there are four landmasses in total and all of them are connected to an odd number of bridges.

Euler had been able to explain why it was impossible to cross each one of Königsberg's bridges once and only once, and furthermore he had generated a rule which could be applied to any network of bridges in any city in the world.

Tool box.

A landmass   /   whimsical   /   a failure   /   a bank

Now test yourself.
  1) *What did the citizens of Königsberg wonder ?*
  2) *Describe how Euler represented the city.*
  3) *What did he notice ? How did he solve that problem ?*

Utilisation de logiciels de mathématiques en langue étrangère

De très nombreux logiciels scientifiques ont été conçus pour des utilisateurs comprenant l'anglais : les commandes, les fichiers d'aide ainsi que les didacticiels sont en anglais.

C'est l'occasion de montrer concrètement aux élèves l'importance de la maîtrise de l'anglais scientifique dans le domaine de l'informatique.

De plus, certains de ces logiciels sont libres et exploitables dès la classe de seconde. Ils permettent d'obtenir rapidement des résultats spectaculaires en mathématiques : là encore, il est aisé de susciter la prise de parole en anglais grâce à l'utilisation de ces logiciels.

Trois logiciels sont évoqués ici : Maple (attention , il ne s'agit pas d'un logiciel libre !)
Geogebra (logiciel libre de géométrie dynamique)
Blender (logiciel libre de modélisation en 3D)

1) utilisation de Maple.

Maple est un logiciel professionnel de calcul formel en langue anglaise dont le fichier d'aide est très complet (rédigé également en anglais).

Les applications sont innombrables.

Voici un premier exemple extrait de la rubrique *Maple new user's tour*.

Maple is a very powerful algebraic calculator that can manipulate and find solutions to many problems symbolically. Being able to use variables instead of specifying numbers at the beginning of a problem allows you to ask "What if...?" types of questions.

Use the **restart** command to clear Maple's internal memory and get started with this page of the tour.
> **restart;**

Working with Expressions

Maple provides different ways of manipulating and displaying expressions to make them easier to verify, or more effective to use. This flexibility allows you to do such things as expand binomials, factor results, simplify trigonometric expressions, assign results to variable names, and convert expressions to different forms.

Expanding and Factoring Expressions
Maple can expand binomials such as $(x + y)^{15}$. The following Maple commands create the expression, and then expand it.
> **expr := (x+y)^15;**
$$expr := (x + y)^{15}$$

> **expand(expr);**

$$x^{15} + 15\,y\,x^{14} + 105\,y^2\,x^{13} + 455\,y^3\,x^{12} + 1365\,y^4\,x^{11} + 3003\,y^5\,x^{10} + 5005\,y^6\,x^9$$
$$+\ 6435\,y^7\,x^8 + 6435\,y^8\,x^7 + 5005\,y^9\,x^6 + 3003\,y^{10}\,x^5 + 1365\,y^{11}\,x^4 + 455\,y^{12}\,x^3$$
$$+\ 105\,y^{13}\,x^2 + 15\,y^{14}\,x + y^{15}$$

After viewing the results, you can use the **factor** command to factor the last result and check the computation.

```
> factor(%);
```
$$(x + y)^{15}$$

C'est l'occasion pour les élèves d'utiliser le vocabulaire spécifique tout en découvrant le principe de fonctionnement d'un logiciel de calcul formel .

L'utilisation de ce logiciel a également l'avantage de montrer aux élèves que la maîtrise de l'anglais scientifique est un atout dans la vie professionnelle : cette activité donne du sens à l'enseignement dispensé en Section Européenne.

Voici un second exemple faisant intervenir de grands entiers.

Computations with Integers

At its most basic level, you can use Maple as a very powerful calculator.
To calculate $(32)(\,12^{13}\,)$, you would enter the following:

```
> 32*12^13;
```
$$3423782572130304$$

Maple recognizes many special operators, including factorial, greatest common divisor, and least common multiple. The following line is an example of the factorial operator in use.

```
> 200!;
```
```
 78865786736479050355236321393218506229513597768717326329474253324    \
     43594499634033429203042840119846239041772121389196388302576      \
     2790242637105061926624952829931113462857270763317237396988943    \
     9224456214516642402540332918641312742829485327752424240757390    \
     3240321257405579568660226031904170324062351700858796178922222    \
     7896237038973747200000000000000000000000000000000000000000000    \
     000000
```

You can easily include the previous expansion of $200!$ in any subsequent calculation without having to type it. The ditto operator, represented by a percent sign ($\%$), refers to the last expression computed by Maple. (For more information on the ditto operator, see the help page for ditto.) The command **ifactor** factors the previous result into its prime factors.

```
> ifactor(%);
```
$$(2)^{197}\ (3)^{97}\ (5)^{49}\ (7)^{32}\ (11)^{19}\ (13)^{16}\ (17)^{11}\ (19)^{10}\ (23)^8\ (29)^6\ (31)^6$$
$$(37)^5\ (41)^4\ (43)^4\ (47)^4\ (53)^3\ (59)^3\ (61)^3\ (67)^2\ (71)^2\ (73)^2\ (79)^2$$
$$(83)^2\ (89)^2\ (97)^2\ (101)\ (103)\ (107)\ (109)\ (113)\ (127)\ (131)\ (137)$$
$$(139)\ (149)\ (151)\ (157)\ (163)\ (167)\ (173)\ (179)\ (181)\ (191)\ (193)$$
$$(197)\ (199)$$

The next command calculates the product again, which is precisely the value of 200!.

```
> expand(%);
 78865786736479050355236321393218506229513597768717326329474253320 \
     443594499634033429203042840119846239041772121389196388302576420 \
     27902426371050619266249528299311134628572707633172373969889430 \
     92244562145166424025403329186413122742829485327752424240757390 \
     03240321257405579568660226031904170324062351700858796178922220 \
     27896237038973747200000000000000000000000000000000000000000000 \
     000000
```

Floating-point Arithmetic

A principal strength of Maple is its ability to do exact arithmetic. Fractions and radicals during computation are not converted to their decimal equivalent, thereby avoiding round-off errors. When you do need to use decimal equivalents, Maple has a command that approximates the value of the expression in a floating-point form.

Consider the expression $\dfrac{2^{30}\sqrt{3}}{3^{20}}$, which is entered on the Maple command line as follows:

```
> (2^30/3^20)*sqrt(3);
```

$$\frac{1073741824\ \sqrt{3}}{3486784401}$$

Use the **evalf** command to generate an approximation in the form of a floating-point value.

```
> evalf(%);
```

$$0.5333783739$$

**2) utilisation de Geogebra**. (http://www.geogebra.org/cms/ )


Geogebra est un logiciel libre de géométrie dynamique disponible dans de nombreuses langues.

Exemple d'activité au niveau de la seconde : une figure tirée d'un énoncé est proposée à l'élève. L'élève doit la reconstruire à l'aide du logiciel.

Tous les menus déroulants sont dans la langue choisie, ce qui permet à l'élève d'utiliser naturellement le vocabulaire spécifique.
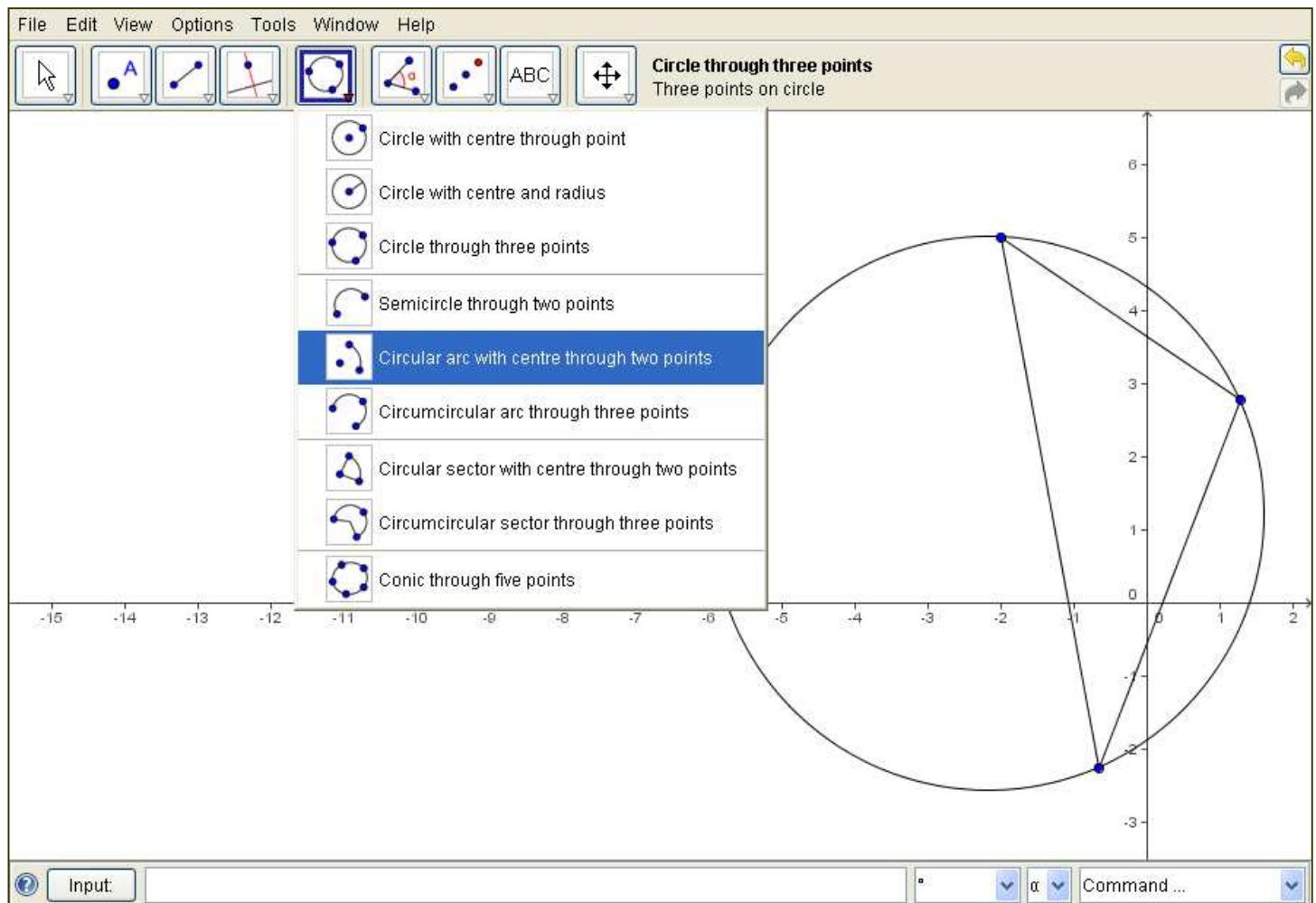


La prise en main de ce logiciel est particulièrement simple.
De nombreux didacticiels sont proposés.

3) **utilisation de Blender**          (http://biorust.com/tutorials/detail/83/us/)
                                (http://www.blender.org/tutorials-help/tutorials/getting-started/)


Blender est un logiciel libre en langue anglaise permettant de modéliser en trois dimensions un objet puis éventuellement de l'animer.

Contrairement à Geogebra, la prise en main de ce logiciel n'est pas immédiate et nécessite un peu de pratique.

En revanche, de nombreux didacticiels gratuits sont disponibles et permettent de mettre en pratique les connaissances des élèves en anglais tout comme en géométrie dans l'espace.

En voici un exemple.

This tutorial will teach you how to make a simple winter scene, including some fir trees and a snowman. It is supposed to show you how to use the most common Blender functionalities (i.e. modelling, adding materials and textures) and shortcuts.
When a shortcut is used for the first time, I'll say something like "press the x key". When I use it again, I'll just put something like "(x)".

## The Snowman
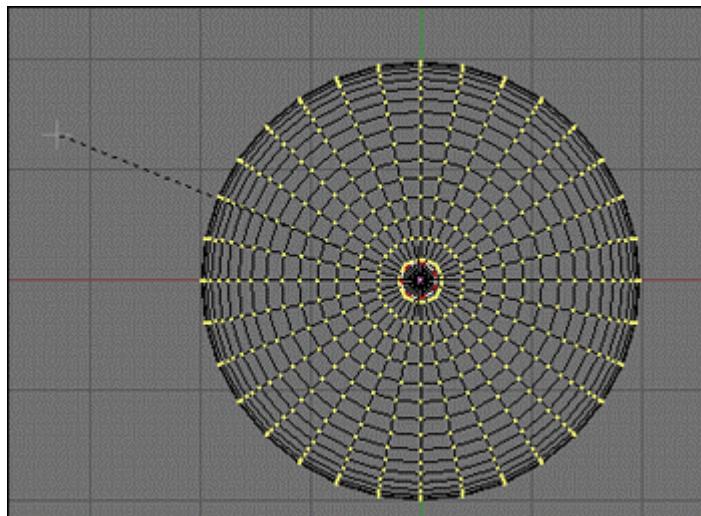
**Step 1 -** Press the **X** key to **delete** the square that is in the middle of the scene. Press the space bar. Select **Add > Mesh > UVSphere** to add a sphere in the scene (click on the OK button for the number of segments and rings, 32 is perfect). **Zoom in** ("**+**" key or the mouse scroll button) and press the **S key (size)** to make the sphere 4 squares wide. (if you want it to be exactly 4 square wide, hold the Ctrl key while you move the mouse ). This is going to be the body of the snowman.
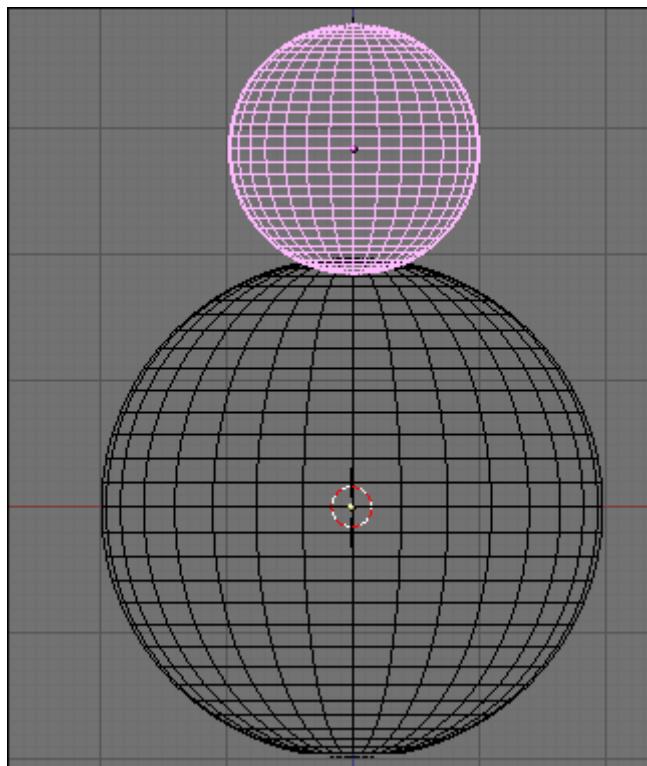
**Step 3 -** Now switch to the **front view** (numpad **1**), and add a 2nd UVsphere in the scene. Resize it (S) to 2 squares wide. Switch again to the object mode (Tab) (when you create an object, blender automatically switches to the Edit Mode). Name it *SnowmanHead* (as done in the previous stage). Now press the **G** key (G=**Grab**) and move the head on top of the body. This is the beginning of the snowman. (You can add some "arms" later if you want, or a 2nd snowball for the body). If you're satisfied with the shape of your snowman, you can "link/join" all the elements together. **Select** them by pressing the **B** key and drawing a square all around them. Then press **Ctrl+J** and click on "**join** selected meshes".
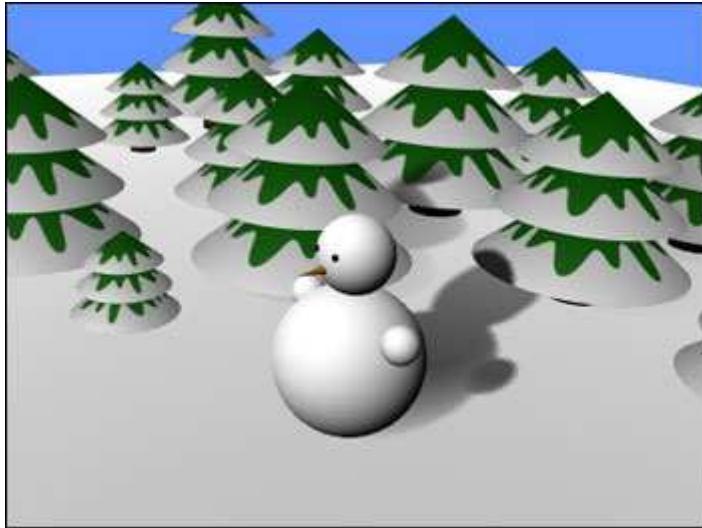
**Step 26 -** Here is what it should look like :)

You'll find the Blender source of this scene below. I added some fancy stuff on the last layer (the one on the bottom right) so you can try some renderings with the 3 layers + this one.

# L'évaluation en Section Européenne

On évalue avant tout la maîtrise du vocabulaire spécifique et l'aptitude à argumenter : le niveau mathématique ne doit pas faire obstacle : le cours de Section Européenne n'est pas un renforcement en mathématiques.

Les capacités orales de communication sont au cœur de cet enseignement, aussi parait-il naturel d'évaluer principalement les élèves à l'oral, même si le recours à une évaluation écrite régulière reste nécessaire.

### a)  L'évaluation en classe de Seconde.

On peut procéder à une brève interrogation orale au début de chaque cours.
En classe de Seconde, cette interrogation orale reste très guidée. On n'hésite pas à intervenir pour aider l'élève, à passer à la question suivante si ce dernier ne parvient pas à répondre. On peut éventuellement demander à un élève de résoudre un exercice déjà traité.
On juge alors de sa maîtrise des définitions et des propriétés vues lors du cours précédent, en relativisant l'importance des fautes de grammaire et de prononciations.
Une évaluation écrite trimestrielle vient compléter ces interrogations orales et permet de faire le point des connaissances acquises.

### b)  En classe de Première et de Terminale.

On peut toujours procéder à une interrogation orale au début de chaque cours, mais celle-ci change d'optique : il s'agit de préparer l'épreuve spécifique du baccalauréat.
Les questions posées sont plus ouvertes. On peut proposer un exercice s'inspirant d'un énoncé déjà traité. On peut éventuellement demander à l'élève de résumer les idées développées dans un texte étudié auparavant.
L'autonomie, la prise d'initiative et l'aisance à l'oral sont valorisées.
Dés la classe de première, l'évaluation écrite trimestrielle est l'occasion de proposer de premiers sujets dans l'esprit de l'épreuve spécifique : le jour de l'épreuve, l'élève dispose de vingt minutes de préparation à l'écrit pour résoudre un exercice.
L'organisation d'un bac blanc oral dans les conditions exactes de l'épreuve d'examen permet à l'élève de parfaire sa préparation : l'élève a face à lui deux examinateurs pour un entretien de près de vingt minutes. L'organisation d'un tel bac blanc nécessite de travailler en collaboration avec un professeur de langue vivante .

### c)  L'épreuve spécifique du baccalauréat : quelques exemples de sujets.

*Optimization*

*Example 1*

Suppose I have a piece of wire of 40 cm long.
If  I make a rectangle from the wire, how should I choose the length and breadth to give me the greatest possible area  ?

*Example 2*

A sheet of cardboard 3ft. by 4 ft. will be made into an open box by cutting equal-sized squares from each corner and folding up the four edges.
What will be the dimensions of the box with largest volume ?

*Example 3*

A zoo keeper plans to build a rectangular cage for her newly arrived panda bears.
The front of the cage is to be made of glass at a cost of $100 per foot and the other three sides of the cage are to be made of concrete costing $50 per foot.
If the zoo keeper has $3,000 to spend on the cage, what dimensions should the cage be so as to maximize the area of the cage ?

## *Probabilites*

*Example 4*

In a certain city, 8% of all mature males have diabetes. The local health service is correctly able to diagnose 95% of those who have the disease, and incorrectly diagnoses 2% of those who don't have diabetes.
1) Find the probability that a randomly selected mature male will be diagnosed as having the disease diabetes.
2) What's the probability that a mature male who is diagnosed with diabetes actually has it ?

## *Sequences*

*Example 5*

A company wants to recruit people to work for a ten month period.
They offer two different types of pay scheme.
A) Starting salary of £1,000 per month, increasing by £100 per month.
B) Starting salary of £900 per month, increasing by 10% per month.
Which pay scheme is the more interesting ?

# Un exemple de progression

**A) La classe de seconde.**

        1) *Sélection de notions mathématiques.*

Il s'agit d'une année d'imprégnation et le niveau mathématique ne doit pas être un obstacle. On privilégie les thèmes mathématiques $3^{i\text{ème}}$ - $2^{\text{nde}}$ , le raisonnement et la description.

Notions traitées :

**-Geometrical constructions.**
 droites remarquables, construction à la règle et au compas.
**-Transformations.**
 réflexions, symétries et rotations
**-Statistics.**
 vocabulaire de base, détermination graphique de la médiane.
**-Functions.**
 vocabulaire de base, lecture graphique des variations, des image et des antécédents.
**-Percentages.**
 dans un contexte économique.
**-Arithmetic.**
 nombres premiers, PGCD, PPCM.

        2) *sélection de thèmes mathématiques.*

**-The history of secret writing.**
**-The value added tax.**
**-Dangers of visual misrepresentation in statistics.**

        3) *renforcement du vocabulaire.*

-L'expression des dimensions ( longueur, surface, volume, temps )
-L'utilisation du comparatif ( supériorité, infériorité ) .
-L'expression du but.

**B) La classe de première.**

        1) *Sélection de notions mathématiques.*

Renforcement des compétences de base et début de la préparation à l'examen.

Notions traitées :

**-Probabilities.**
 vocabulaire de base, utilisation d'arbres.
**-Sequences.**
 vocabulaire de base.

***-Differentiation.***
nombre dérivé, tangente et variations.
***-Statistics.***
Définition et détermination graphique des quartiles, écart type.

2) *sélection de thèmes mathématiques.*

***-Fermat's last theorem.***
***-The evolution of number theory.***

3) *renforcement du vocabulaire.*

-Expression de l'objectivité ( passif )
-Expression de la fréquence

## C) La classe de terminale.

1) *Sélection de notions mathématiques.*

Développement de l'autonomie à l'oral et résolution de problèmes ouverts.

Notions traitées :

***-Optimization.***
Problèmes de maximum et de minimum.
***-Conditional probabilities.***
Indépendance, formule des probabilités totales.
***-Sequences.***
somme des premiers termes.

2) *sélection de thèmes mathématiques.*

***-Graph theory.***
***-The proof  by contradiction.***
***-Zero in modern mathematics.***

3) *renforcement du vocabulaire.*

-Exprimer la cause et la conséquence.
-Articuler un raisonnement.